## Electrical Technology (EE-101-F)

## Contents

$>$ Series \& Parallel Combinations > KVL\&KCL
> Introduction to Loop \& Mesh Analysis Frequently Asked Questions
NPTEL Link

## Series-Parallel Resistances



There are three series sections in this circuit; sections 1 and 2 are parallel banks.

## Series and Parallel Circuits

- Series Circuit
- Current is the same in all components.
- $V$ across each series $R$ is $I \times R$.
$-V_{T}=V_{1}+V_{2}+V_{3}+\ldots+$ etc.
- Parallel Circuit
- Voltage is the same across all branches.
$-I$ in each branch $R$ is $V / R$.
$-I_{T}=I_{1}+I_{2}+I_{3}+\ldots+$ etc.


## Series-Parallel Circuits

## - Example:

- Find all currents and voltages in the circuit given below
- Step 1: Find $R_{T}$.
- Step 2: Calculate main line current as $I_{T}=V_{\substack{R_{G} \\ R_{1} \\ R_{1} \\ 15 \\ \hline}} / R_{T}$

(a)

(b)
: Reducing a series-parallel circuit to an equivalent series circuit to find the $R T$. (a) Actual circuit. (b) $R_{3}$ and $R_{4}$ in parallel combined for the equivalent $R_{T}$.

(c)

$R_{2}=$
$30 \Omega$
(d)

Fig (c) $R_{T}$ and $R_{6}$ in series added for $R_{13 .}$ (d) $R_{13}$ and $R_{5}$ in parallel combined for $R_{18}$.

(e)

Fig. e: The $R_{18}, R_{1}$, and $R_{2}$ in series are added for the total resistance of $50 \Omega$ for $R_{T}$.

## Analyzing Series-Parallel Circuits with Random Unknowns

- In solving such circuits, apply the same principles as before:
- Reduce the circuit to its simplest possible form.
- Apply Ohm's Law.


## Voltage divider rule

The voltage drop across any given resistor in a series circuit is equal to the ratio of that resistor to the total resistance, multiplied by source voltage.

Assume $R_{1}$ is twice the size of $R_{2}$. What is the voltage across $R_{1}$ ? 8 V


## Voltage divider

Voltage dividers can be set up for a variable output using a potentiometer. In the circuit shown, the output voltage is variable.

What is the largest output voltage available? 5.0 V


Power in Series Circuits

## Exalu|l|

Use the voltage divider rule to find $V_{1}$ and $V_{2}$. Then find the power in $R_{1}$ and $R_{2}$ and $P_{T}$.

Applying the voltage divider rule:

$$
\begin{aligned}
& V_{1}=20 \mathrm{~V}\left(\frac{470 \Omega}{800 \Omega}\right)=11.75 \mathrm{~V} \\
& V_{2}=20 \mathrm{~V}\left(\frac{330 \Omega}{800 \Omega}\right)=8.25 \mathrm{~V}
\end{aligned}
$$



The power dissipated by each resistor is:

$$
\left.\begin{array}{l}
P_{1}=\frac{(11.75 \mathrm{~V})^{2}}{470 \Omega}=0.29 \mathrm{~W} \\
P_{2}=\frac{(8.25 \mathrm{~V})^{2}}{330 \Omega}=0.21 \mathrm{~W}
\end{array}\right\} \begin{aligned}
& P_{\mathrm{T}}= \\
& 0.5 \mathrm{~W}
\end{aligned}
$$

## Circuit Ground

The term "ground" typically means a common or reference point in the circuit.

Voltages that are given with respect to ground are shown with a single subscript. For example, $V_{\mathrm{A}}$ means the voltage at point A with respect to ground. $V_{\mathrm{B}}$ means the voltage at point B with respect to ground. $V_{\mathrm{AB}}$ means the voltage between points A and B .
QLIEStiOn What are $V_{\mathrm{A}}, V_{\mathrm{B}}$, and $V_{\mathrm{AB}}$ for the circuit shown?

$$
V_{\mathrm{A}}=12 \mathrm{~V} \quad V_{\mathrm{B}}=8 \mathrm{~V} \quad V_{\mathrm{AB}}=4 \mathrm{~V}
$$

## Key Terms

Circuit ground A method of grounding whereby the metal chassis that houses the assembly or a large conductive area on a printed circuit board is used as a common or reference point; also called chassis ground.

Kirchhoff's A law stating that (1) the sum of the voltage voltage law drops around a closed loop equals the source voltage in that loop or (2) the algebraic sum of all of the voltages (drops and source) is zero.

Open A circuit condition in which the current path is broken.

## Key Terms

Series In an electric circuit, a relationship of components in which the components are connected such that they provide a single path between two points.

Short A circuit condition in which there is zero or an abnormally low resistance between two points; usually an inadvertent condition.

Voltage divider A circuit consisting of series resistors across which one or more output voltages are taken.

## OHM'S LAW

- Ohm's law states that the voltage across a resistor is directly proportional to the current I flowing through the resistor.
- •Mathematical expression for Ohm's Lawis as follows: $R=$ Resistance
- Two extreme possible values of R:
- 0 (zero) and $\infty$ (infinite)
- are related with two basic circuit concep
- short circuit and open circuit.


## Branches, Nodes, Loops

- A branch represents a single element such as a voltage source or a resistor.
- A node is the point of connection between two or more branches.
- A loop is any closed path in a circuit.
- A network with b branches, n nodes, and I independent loops will satisfy the fundamental theorem of network topology:
- $b=1+n-1$

Original Circuit


Network schematics or graph


How many branches, nodes and loops are there?

## Kirchhoff's Current Law

- Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.
- $\Sigma I=0$



## Kirchhoff's Voltage Law

Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.
$\Sigma \mathrm{Vi}=0$


Kirchhoff's Voltage Law:

- Example

Applying the KVL equation for the circuit of the figure below.


$$
\begin{aligned}
& \text { va- v1- vb- v2- v3 = } 0 \text { V1 = } \\
& I \cdot R 1 ; v 2=I \cdot R 2 ; \\
& v 3=I \cdot R 3 \\
& v a-v b=I \cdot(R 1+R 2+R 3) \\
& \text { I= (Va- Vb)/ (R1 + R2 + } \\
& \text { R3) }
\end{aligned}
$$

## KIRCHHOFF VOLTAGE LAW

# ONE OF THE FUNDAMENTAL CONSERVATI ON LAWS I N ELECTRI CAL ENG NERI NG 

THI S IS A CONSERVATI ON OF ENERGY PRI NCI PLE "ENERGY CANNOT BE CREATED NOR DESTROYED"

## KI RCHHOFF VOLTAGE LAW (KVL)

KVL IS A CONSERVATI ON OF ENERGY PRI NCI PLE A POSI TI VE CHARGE GAI NS ENERG AS IT MDVES TO A PO NT W TH H GHER VOLTAGE AND RELEASES ENERG IF IT MDVES TO A PO NT WTH LOWER VOLTAGE

$$
\begin{aligned}
& \Delta W=q\left(V_{B}-V_{A}\right) \\
& \\
& \\
& \stackrel{q}{\circ} \oplus \\
& { }^{\circ} V_{B} \\
& V_{A}
\end{aligned}
$$



A "THOUGTT EXPERI MENT"


If THE CHARGE COMES BACK TO THE SAME I NI TI AL POI NT THE NET ENERGY GAI N MUST BE ZERO (Conservati ve net work)
OTHERW SE THE CHARGE COULD END UP WTH I NFI NI TE ENERG, OR SUPPLY AN I NFI NI TE AMDUNT OF ENERG

$$
\boldsymbol{q}\left(\boldsymbol{V}_{A B}+\boldsymbol{V}_{B C}+\boldsymbol{V}_{C D}\right)=0
$$

KVL: THE ALGEBRAI C SUM OF VOLTAGE DROPS AROUND ANY LOOP MUST BE ZERO

$$
\begin{gathered}
\mathrm{O}-V+\bigcirc \equiv \mathrm{O} \equiv \mathrm{O}+(-V)-\bigcirc \\
A \quad A \\
\text { A VOLTAGE RISE IS } \\
\text { A NEGATIVE DROP }
\end{gathered}
$$

## KVL (Kirchhoff's Voltage Law)

The sum of the potential differences around a closed loop equals zero.

Sum of the Voltage drops across resistors equals the Supply Voltage in a Loop.

## Voltages from Mesh Currents



$$
V_{R}=I_{1} R
$$

$$
V_{R}=\left(I_{1}-I_{2}\right) R
$$

## Example: KVL Around Mesh 1



$$
\begin{gathered}
-V_{1}+I_{1} 1 \mathrm{k} \Omega+\left(I_{1}-I_{2}\right) 1 \mathrm{k} \Omega=0 \\
I_{1} 1 \mathrm{k} \Omega+\left(I_{1}-I_{2}\right) 1 \mathrm{k} \Omega=V_{1}
\end{gathered}
$$

## KVL

## Around Mesh 2



$$
\begin{gathered}
\left(I_{2}-I_{1}\right) 1 \mathrm{k} \Omega+I_{2} 1 \mathrm{k} \Omega+V_{2}=0 \\
\left(I_{2}-I_{1}\right) 1 \mathrm{k} \Omega+I_{2} 1 \mathrm{k} \Omega=-V_{2}
\end{gathered}
$$

## Steps of Mesh Analysis

1. Identify mesh (loops).
2. Assign a current to each mesh.
3. Apply KVL around each loop to get an equation in terms of the loop currents.
4. Solve the resulting system of linear equations.

## Matrix Notation

- The two equations can be combined into a single matrix/vector equation.

$$
\left[\begin{array}{cc}
1 \mathrm{k} \Omega+1 \mathrm{k} \Omega & -1 \mathrm{k} \Omega \\
-1 \mathrm{k} \Omega & 1 \mathrm{k} \Omega+1 \mathrm{k} \Omega
\end{array}\right]\left[\begin{array}{c}
I_{1} \\
I_{2}
\end{array}\right]=\left[\begin{array}{c}
V_{1} \\
-V_{2}
\end{array}\right]
$$

## Cramer's Rule

$$
\begin{aligned}
{\left[\begin{array}{c}
E_{1} \\
-E_{2}
\end{array}\right] } & =\left[\begin{array}{c}
I_{1} \\
I_{2}
\end{array}\right]\left[\begin{array}{cc}
\left(R_{1}+R_{2}\right) & \left(-R_{3}\right) \\
\left(-R_{3}\right) & \left(R_{2}+R_{3}\right)
\end{array}\right] \\
I_{1} & =\frac{\left[\begin{array}{cc}
E_{1} & \left(-R_{3}\right) \\
-E_{2}\left(R_{2}+R_{3}\right)
\end{array}\right]}{\left[\begin{array}{cc}
\left(R_{1}+R_{2}\right) & \left(-R_{3}\right) \\
\left(-R_{3}\right) & \left(R_{2}+R_{3}\right)
\end{array}\right]}
\end{aligned}
$$

$$
I_{2}=\frac{\left[\begin{array}{cc}
\left(R_{1}+R_{2}\right) & E_{1} \\
\left(-R_{3}\right) & -E_{2}
\end{array}\right]}{\left[\begin{array}{cc}
\left(R_{1}+R_{2}\right) & \left(-R_{3}\right) \\
\left(-R_{3}\right) & \left(R_{2}+R_{3}\right)
\end{array}\right]}
$$

## Another Example



## 1. Identify Meshes



## 2. Assign Mesh Currents



## Current Sources

- The current sources in this circuit will have whatever voltage is necessary to make the current correct.
- We can't use KVL around any mesh because we don't know the voltage for the current sources.
- What to do?


## Current Sources

- The 4 mA current source sets $I_{2}$ :

$$
I_{2}=-4 \mathrm{~mA}
$$

- The $2 m A$ current source sets a constraint on $I_{1}$ and $I_{3}$ :

$$
I_{1}-I_{3}=2 \mathrm{~mA}
$$

- We have two equations and three unknowns. Where is the third equation?


## Supermesh



